Zero-length springs and slinky coils – Marking Scheme

Part A: Statics

A.1	First method:	
	realized that $\frac{\Delta y}{\Delta l} = \frac{L}{L_0}$	0.2 pts
	realized that $L = \frac{F}{k}$	0.1 pts
	mentioned that $\Delta y = \Delta l$ if $F < kL_0$	0.1 pts
	correct result	0.1 pts
	Second method:	
	realized $k^* = k\frac{L_0}{\Delta l}$	0.2 pts
	mentioned that $\Delta y = \Delta l$ if $F < kL_0$	0.1 pts
	used $F - F_0 = k^*(\Delta y - \Delta l)$	0.1 pts
	correct result	0.1 pts
A.2	used $F(x) = \frac{kL_0}{\Delta l}x$	0.1 pts
	used $\Delta W = \int_{\Delta l}^{\Delta l_2} \frac{kL_0}{\Delta l} x dx$ (if $\Delta W = \frac{kL_0}{\Delta l}(\Delta y - \Delta l)^2$ – no pts.)	0.3 pts
	correct result	0.1 pts
A.3	calculating l_0, the length of the closed turns	0.2 pts
	correct result for l_0	0.3 pts
	realized $F(l) = Mg\frac{l}{L_0}$	0.3 pts

	expressed $dy = \frac{Mg}{kL_0^2} l dl$	0.2 pts
	used $H = l_0 + \int_{l_0}^{L_0} \frac{Mg}{kL_0^2} l dl$ (if assumed $l_0 = 0$: 0.1 pts)	0.3 pts
	correct result expressed using the required variables (if assumed $l_0 = 0$: 0.4 pts) correct result but not expressed using the required variables (if assumed $l_0 = 0$: 0.1 pts)	0.7 pts 0.4 pts

Part B: Dynamics

B.1	used $H(l) = \frac{l^2}{2l_0} + \frac{l_0}{2}$	0.3 pts
	the contribution of element dl to the center of mass: $\left(\frac{l^2}{2l_0} + \frac{l_0}{2}\right)\frac{Mdl}{L_0}$	0.2 pts
	$H_{cm} = \frac{1}{M}\left[\frac{l_0}{2}\alpha M + \int_{l_0}^{L_0} H(l) dm\right]$ (first term 0.3, second term 0.3)	0.6 pts
	correct calculation of $H_{cm} = L_0\left[\frac{1}{6\alpha} - \frac{\alpha^2}{6} + \frac{\alpha}{2}\right]$ (if assumed $l_0 = 0$: 0.3 pts)	0.4 pts
	found the displacement of the center of mass: $\Delta H = H_{cm} - \frac{L_0}{2}$	0.2 pts

	used $\frac{g}{2}t_c^2 = H_{\text{cm}} - \frac{L_0}{2}$	0.2 pts
	correct result expressed using the required variables correct result but not expressed using the required variables	0.5 pts 0.4 pts
	numerical value of time	0.1 pts
	Another method: Use A,B from B.2 and write $$t = \int_{l_0}^{L_0} \frac{dH(l)}{\sqrt{Al+B}}$$ 0.4 pts – both boundaries of integration 0.4 pts – $dH(l)$, dl – no pts 0.2 pts - $1/v$ 1.4 pts if correct computation of integration leading to a correct result expressed with required variables, 1.3 pts expressed with wrong variables	
B.2	Default method – work of external force equals the change in kinetic energy	
	calculated the position of the upper part ($l < l' < L_0$) before the spring is released, used $H(l') = \frac{l'^2}{2l_0} + \frac{l_0}{2}$	0.2 pts
	$H_{cm-upper} = \frac{L_0}{M(L_0 - l)} \int_l^{L_0} H(l')dm = \frac{L_0}{M(L_0 - l)} \int_l^{L_0} \left(\frac{l'^2}{2l_0} + \frac{l_0}{2} \right) \frac{Mdl'}{L_0}$ if using M instead of $\frac{M(L_0 - l)}{L_0}$: only 0.1 pts	0.3 pts
	found $H_{cm-upper} = \frac{L_0^2 + L_0 l + l^2}{6l_0} + \frac{l_0}{2}$	0.5 pts

	found the position of the center of mass of the moving part: $$H_{cm-upper-f}=\frac{l^2}{2l_0}+\frac{l_0}{2}+\frac{1}{2}(L_0-l)$$ (first two terms – 0.1 pts, third term – 0.2 pts)	0.3 pts
	$$\Delta H_{cm-upper}=H_{cm-upper}-H_{cm-upper-f}=\frac{(L_0-l)(L_0+2l)}{6l_0}-\frac{1}{2}(L_0-l)$$ (finding the difference – 0.1 pts, result for the difference – 0.2 pts)	0.3 pts
	understood that the external force is Mg (any other expression – 0 pts)	0.3 pts
	Using $Mg\Delta H_{cm-upper}=\frac{M(L_0-l)}{2L_0}v^2$	0.3 pts
	obtained $v^2_{upper-f}=\frac{2g}{3\alpha}l+\frac{L_0 g}{3\alpha}-L_0 g$	0.3 pts
	Alternative method, using momentum and impulse on part I:	
	Realized that that $F=Mg$	0.3 pts
	Wrote that $Fdt=dp$	0.3 pts
	Realized that $dp=mdv+vdm$ (0.3 pts per term)	0.6 pts
	Realized that $vdt=dy-dl$ (0 pts if $vdt=dy$)	0.4 pts
	Getting a differential equation for v^2	0.3 pts
	Substituting $v^2=Al+B$	0.2 pts
	Comparing coefficients to get A, B (0.2 pts each)	0.4 pts
	Other methods: There are other ways to solve this part Finding A and B without deriving $v_{\mathrm{I}}(l)=\sqrt{Al+B}$	1.3 pts
B.3	realized that the velocity decreases in time when a stationary part still exists	0.1 pts

	in case a student gets $A < 0$: 0.1 pts for the opposite conclusion	
	substituted $l = l_0$ to find $v_1(l_0) = \sqrt{Al_0 + B}$	0.1 pts
	used conservation of momentum (plastic collision) to find correct result $$v_{min} = \frac{m_{top} v(l_0)}{M}$$	0.3 pts

Part C: Energetics

C.1	an equation relating Q and W (without gravitational energy)	0.4 pts
	use $dW = \frac{kL_0}{2dl}\left(\left(\frac{1}{l_0} l dl\right)^2 - dl^2\right) = \frac{kL_0}{2}\left(\frac{l^2}{l_0^2} - 1\right) dl$	0.4 pts
	calculated the elastic energy of the spring: $W = \int_{l_0}^{L_0} \frac{kL_0}{2}\left(\frac{l^2}{l_0^2} - 1\right) dl$	0.4 pts
	correct result $W = MgL_0 \frac{(1-\alpha)^2 (2\alpha + 1)}{6\alpha}$	0.3 pts