Solution / marking scheme – Neutron Stars (10 points)

General rules
- In the following, "coefficients" refer to the numerical factors and do not include parameters.

Part A. Mass and stability of nuclei (2.5 points)

A.1 (total 0.9 pt)

(0.9 pt)

$A = 50$
- No reduction if $A = 5.0 \times 10^1$.
- 0.8 pt if the value is in the range 49.5–50.4.

partial points

(0.2 pt) $$\frac{B}{A} = a_V - a_S A^{-1/3} - \frac{a_C}{4} A^{2/3} \tag{A.1.1}$$

- No reduction if the difference from (A.1.1) is only the overall coefficient. This rule is applied throughout.

(0.1 pt) $$\frac{d(B/A)}{dA} = 0 \tag{A.1.2}$$

(0.2 pt) $$\frac{a_S}{3} A^{-4/3} - \frac{a_C}{6} A^{-1/3} = 0 \tag{A.1.3}$$

- Points for (A.1.2) are given if (A.1.3) is stated although (A.1.2) is not explicitly written.

(0.2 pt) $$A = \frac{2a_S}{a_C} \tag{A.1.4}$$

- 0.7 pt is given if the correct expression for A appears even if the intermediate steps are not fully written.

A.2 (total 0.9 pt)

(0.9 pt)

$Z^* = 79$

- No reduction if $Z^* = 78$.
- 0.8 pt if the value is in the range 77.5–79.4.

partial points

(0.3 pt) $$-2a_C \frac{Z^*}{A^{1/3}} - 4a_{\text{sym}} \frac{2Z^* - A}{A} = 0 \tag{A.2.1}$$

(0.4 pt) $$Z^* = \frac{1}{1 + \dfrac{a_C}{4a_{\text{sym}}} A^{2/3}} \cdot \frac{A}{2} \tag{A.2.2}$$

- No reduction if $a_C/4a_{\text{sym}}$ is replaced by the numerical value in the range 0.007–0.008.

A.3 (total 0.7 pt)

(0.7 pt)

$C_{\text{fission}} = 0.70$

- No reduction if $C_{\text{fission}} = 0.7$.

partial points

(0.3 pt) $$a_S \left[A^{2/3} - 2\left(\frac{A}{2}\right)^{2/3} \right] + a_C \left[\frac{Z^2}{A^{1/3}} - 2\frac{(Z/2)^2}{(A/2)^{1/3}} \right] > 0 \tag{A.3.1}$$

- No point if a_V is not canceled.

(0.2 pt) $$\frac{Z^2}{A} > \frac{2^{1/3} - 1}{1 - 2^{-2/3}} \cdot \frac{a_S}{a_C} \tag{A.3.2}$$

- Points for (A.3.1) are given if (A.3.2) is stated although (A.3.1) is not explicitly written.
- The coefficient may have different expressions, e.g., with $x = 2^{1/3}$,

$$\frac{x-1}{1-x^{-2}} = \frac{x^2}{1+x} = \frac{x}{1+x^{-1}} = \cdots = 0.702414\ldots$$

Part B. Neutron star as a gigantic nucleus (1.5 points)

B.1 (total 1.5 pt)

(0.8 pt)

$a_{\mathrm{grav}} = 6 \times 10^{-37}$ MeV

- No reduction if the unit is not written.
- 0.7 pt if only the order of magnitude is correct.

partial points

(0.4 pt) $a_{\mathrm{grav}} = \dfrac{3}{5}\dfrac{Gm_N^2}{R_0}$ (B.1.1)

(0.2 pt) $a_{\mathrm{grav}} = \dfrac{3}{5}\dfrac{\hbar c m_N^2}{R_0 M_P^2}$ (B.1.2)

- Points for (B.1.1) are given if (B.1.2) is stated although (B.1.1) is not explicitly written.
- No reduction if $\hbar$ is mistyped.

(0.7 pt)

$A_c = 4 \times 10^{55}$

- No reduction for $A_c = 5 \times 10^{55}$.
- 0.6 pt if only the order of magnitude is correct.

partial points

(0.2 pt) $a_V A - a_{\mathrm{sym}} A + a_{\mathrm{grav}} A^{5/3} > 0$ (B.1.3)

(0.3 pt) $A_c = \left(\dfrac{a_{\mathrm{sym}} - a_V}{a_{\mathrm{grav}}}\right)^{3/2}$ (B.1.4)

- Points for (B.1.3) are given if (B.1.4) is stated although (B.1.3) is not explicitly written.

Part C. Neutron star in a binary system (6.0 points)

C.1 (total 1.0 pt)

(1.0 pt)

$$\Delta\tau_{\mathrm{II}} = \left(1 - \frac{\Delta\phi}{c^2}\right)\Delta\tau_{\mathrm{I}}$$

- No points if the coefficient is wrong.

partial points

(0.3 pt) $v^2 = 2g\Delta h = 2\Delta\phi$ or $v = \sqrt{2\Delta\phi}$ (C.1.1)

(0.5 pt) $\Delta\tau_{\mathrm{II}} = \sqrt{1 - v^2/c^2}\,\Delta\tau_{\mathrm{I}}$ or $\Delta\tau_{\mathrm{II}} = \sqrt{1 - 2\frac{\Delta\phi}{c^2}}\,\Delta\tau_{\mathrm{I}}$ (C.1.2)

- Points for (C.1.1) are given if (C.1.2) is stated although (C.1.1) is not explicitly written.

C.2 (total 1.8 pt)

(1.8 pt)

$$\Delta t = \frac{2GM_{\mathrm{WD}}}{c^3}\log\left(\frac{4|x_N|x_E}{d^2}\right)$$

- No reduction if 4 is missing in log.
- No reduction if $|x_N|$ is written as $-x_N$.
- 0.1 pt is subtracted if the modulus in $|x_N|$ is missing.
- No points if other coefficients are wrong.

partial points

(0.5 pt) $t_{\text{E-N}} = \displaystyle\int_{x_N}^{x_E} \frac{dx}{c_{\mathrm{eff}}(x)}$ or $\Delta t_{\text{E-N}} = \dfrac{\Delta x}{c_{\mathrm{eff}}(x)}$ (C.2.1)

(0.4 pt) $t_{\text{E-N}} \simeq \dfrac{1}{c}\displaystyle\int_{x_N}^{x_E} dx\left(1 + \frac{2GM_{\mathrm{WD}}}{c^2\sqrt{x^2+d^2}}\right)$ (C.2.2)

- 0.1 pt is subtracted if the coefficient is wrong.

(0.3 pt) $\Delta t = \dfrac{2GM_{\mathrm{WD}}}{c^3}\displaystyle\int_{x_N}^{x_E} \frac{dx}{\sqrt{x^2+d^2}}$ (C.2.3)

(0.3 pt) Inside the logarithm: $\sqrt{x_N^2+d^2} + x_N \simeq \dfrac{d^2}{2|x_N|}$ and $\sqrt{x_E^2+d^2} - x_E \simeq \dfrac{d^2}{2x_E}$ (C.2.4)

C.3 (total 1.8 pt)

(1.8 pt)

$$\Delta t_{\max} - \Delta t_{\min} = \frac{2GM_{\mathrm{WD}}}{c^3} \log(4/\varepsilon^2)$$

- No reduction if log is written as ln.

partial points

(0.6 pt) $$\Delta t_{\max} = \frac{2GM_{\mathrm{WD}}}{c^3} \log(4x_E/L\varepsilon^2) \tag{C.3.1}$$

- No subtraction points if the factor in log is different but consistent with that in C.2.
- 0.1 pt is subtracted if the coefficient is wrong.

(0.2 pt) Because of $x_N > 0$ the approx. in log is changed: $x_N + \sqrt{x_N^2 + d^2} \simeq 2L$ (C.3.2)

(0.4 pt) $$\Delta t_{\min} = \frac{2GM_{\mathrm{WD}}}{c^3} \ln(x_E/L) \tag{C.3.3}$$

- Points for (C.3.2) are given if (C.3.3) is stated although (C.3.2) is not explicitly written.
- 0.1 pt is subtracted if the coefficient is wrong.

(0.3 pt) Points are given if L and x_E dependence is canceled in log. (C.3.4)

C.4 (total 0.8 pt)

(0.8 pt)

$$M_{\mathrm{WD}}/M_\odot = 0.5$$

- No reduction if the value is in the range 0.4–0.5.

partial points

(0.2 pt) $\varepsilon^2 \simeq 2 \times (1 - 0.99989) = 0.00022$ (C.4.1)

(0.2 pt) From the given graph, $\Delta t_{\max} - \Delta t_{\min} \approx 50\,\mu\mathrm{s}$ (C.4.2)

- No reduction if the value from the graph is in the range 40–50 μs.

(0.2 pt) $M_{\mathrm{WD}}/M_\odot \simeq 5/\ln(4/\varepsilon^2)$ (C.4.3)

- No reduction if the numerator is in the range 4–5.

C.5 (total 0.4 pt)

(0.4 pt)

$p = -\dfrac{3}{2}$ or -1.5

- No points if the sign is wrong.

partial points

(0.3 pt) $R^3\omega^2 = (\text{const.})$ (C.5.1)

C.6 (total 0.2 pt)

(0.2 pt)

The most appropriate profile is (b).